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PRODUCTION FUNCTIONS OF FAST BREEDER REACTORS

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Author's Note: This is a first draft of one of four chapters in a study tentatively entitled "An Economic Strategy for Developing the Fast Breeder Reactor" sponsored by Resources For The Future. Although it is in rough form, comments are urgently sought at this point because of its importance for the chapters that follow.

Reactors added to electric generating systems in the 1980's will be governed by technical limits on output with given inputs which can be expressed in "production functions" $MW_t = f(K,F)$ for thermal megawatts of capacity MW_t as a function of the amount of capital equipment K and the inventory of atomic fuel F. The limits are set by the state of the art for any type of reactor: a particular design, using materials with limited conductivity and durability, is capable of delivering only so much heat from atomic fission. Breeder reactors, as a new state of the art, will have new and different production functions in the sense that new combinations of capital and fuel will be available to produce some of the desired levels of capacity. If liquid metal, steam, and gas-cooled fast breeders are all developed, then there will be three new reactors each with a different production function; this would make it possible to obtain any given amount of new capacity from a relatively larger number of additional combinations of capital and fuel.

Some of the additional combinations could be most interesting. One of central interest is the least cost combination: given input factor prices P_K, P_F, some amounts of capital and fuel designated as K^*, F^* produce MW_t^* at least total expenditure. A particular breeder type might attain K^*, F^* for MW_t^* while the others did not. Another breeder might be the only type able to produce MW_t^* for all the K,F combinations that could possibly be least cost combinations (given that capital and fuel prices are not known for certain).

In the 1960's, rather than the 1980's, these combinations can only be known by specifying the 1980 production functions. Of course the functions cannot be specified, but have to be predicted because research breakthroughs

could reduce K for given F and MW_t, but research failures would result in higher values of K than called for in present design studies. To be able to predict these future functions, it must be assumed that the present design studies are indicators of "central tendency" for MW_t, K, and F for 1980 plants -- or that, even though both higher and lower levels of capacity are possible, the design estimate is the best estimate at the present time of plant capability for given amounts of K,F. Then it has to be assumed as well that the functions take some general form. The form here is

$$MW_t^{\delta_1} F_b^{\delta_2} = K^{\beta} F^{\psi} U(K,F)$$

with MW_t the maximum sustainable output of thermal energy at any time during a thirty-year reactor lifetime and F_b the newly bred fuel from capital K and fuel F, subject to sporadic reductions or increases in relative magnitude as shown by the factor U (which also depends on K,F). This definition seems plausible because it implies that the marginal products of capital for either capacity ($\partial MW_t / \partial K$) or breeding ($\partial F_b / \partial K$) depend upon the amounts of the other input and of fuel. Such behavior, according to most of the present design studies, does follow from variations in outlet temperature and in fuel rod design.

Given the assumptions, there is information throughout the current design studies for estimating the parameters of the function. Rewriting the relation so that

$$MW_t = K^{\beta/\delta_1} F^{(\psi-\delta_2)/\delta_1} U(K,F) = K^{\beta'} F^{\psi'} U(K,F)$$

data are required for estimating β' and ψ' for the different types of breeder reactors. The data can be obtained a number of ways. The most readily available sources are chapters on variations in fuel or fuel coolant temperatures and pressures which show the tradeoffs of fuel for capital in the design, and chapters on "scaling up" or "scaling down" the reactor in size. The first shows $\partial K/\partial F\big|_{MW_t=MW_1}$ which is equal to $-\psi'K_1/\beta'F_1$ from differentiating K with respect to F in the production function. The studies of scale can produce values K_1, F_1, MW_1 and K_2, F_2, MW_2 for two sizes of a reactor type that can be inserted in the production function definition to form the equation $\beta'\log(K_1/K_2) + \psi'\log(F_1/F_2) = \log(MW_1/MW_2)$. The two equations provide sufficient detail to solve for β' and ψ'.

The estimates of β' and ψ' are constructed here for each fast breeder type. They are based on values of $K > 0$ and $F > 0$, since these are the only values that minimize costs. They are restricted to $\beta' > 0$, $\psi' > 0$ as well: otherwise the marginal products of K and/or F would be negative, and less of any such factor could be used while producing the same output.

The Liquid Metal Fast Breeder Reactor

The thermal energy reactor, to this point in time, has been based on a fuel core made up of a collection of fuel rods in which uranium or plutonium fission takes place. The core may be cylindrical or even pancaked in shape, and the numbers and sizes of rods vary with core temperature and the means for removing the heat energy. In "fast breeder reactors," as contrasted with "thermal reactors," there is no moderating material added to the core

to slow down the neutrons from fission; as a result, there is reduced probability of continued fissioning of uranium (U^{233}, U^{235}), but increased probability of nonfission capture of neutrons by U^{238} so as to "breed" plutonium Pu^{239}. The bred plutonium is fissionable by the fast neutrons.

Some of the technology will use liquid sodium as the means for transferring fission heat from the core to a steam source for power generation. Sodium is molten at 200°F but does not boil at temperatures below 1600°F, so that a wide range of heat-transfer temperatures is possible at environmental pressures; it has excellent heat absorption characteristics, but it is corrosive to many metals and it reacts so strongly to water that the combination is explosive under pressure. As a result of this volatility, the flow diagram for a sodium-cooled reactor is complex. The heat transfer from the core to the steam generator is divided into a number of independent and self-contained steps so that the failure of sodium flow at one step will not disrupt the entire system. Most components in the separate systems have back-up and by-pass routes, as well, so that any malfunction of the equipment resulting from sodium corrosion is isolated and separated from the continued operation of other components. Typically, the heat from the core is routed by sodium in a primary transfer system to three or more secondary sodium systems, and then transferred from the secondary systems to a number of separate steam generators; each primary or secondary sodium loop is self-contained and the heat exchanger is made up of a shell of one material in which is contained the tube of the second material.

There have been four major design studies of 1980 liquid sodium fast reactors with capacity close to 2500 thermal megawatts, and a number of article-length analyses either of smaller reactors or of particular design variations.[1] Each of the major designs has an elaborate heat transfer system -- in particular, six separate sodium loops before the transfer of heat to two separate steam generators. They place no specific demands on the core or the fuel rod design; the configuration and arrangement of rods, and their

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[1] The four design studies for a large liquid metal fast breeder reactor sought to provide 1,000 electrical megawatts of capacity in each case. Some of the designs involved higher levels of thermal efficiency than others, so that the thermal megawatts of capacity varied around 2500 thermal megawatts. The four studies are:

Allis-Chalmers, et.al., Large Fast Reactor Design Study, (ACNP-64503, January 1964).

Combustion Engineering, Liquid Metal Fast Breeder Reactor Design Study, (CEND-200, January 1964).

The General Electric Company, Liquid Metal Fast Breeder Reactor Design Study (GEAP-4418, January 1964).

The Westinghouse Electric Company, Liquid Metal Fast Breeder Reactor Design Study (WCAP-3251-1, January 1964).

These are referred to as the "major designs"; but there are others of importance or interest as to particular parameters. The study by K. P. Cohen and G. L. O'Neill, "Safety and Economic Characteristics of a 1000 Megawatt Electric Fast Sodium-Cooled Reactor Design," (ANL 6700, 1965) contributes to the evaluation of system reliability. The study by W. Hafele, D. Smidt, and K. Wirtz, "The Karlsruhe Reference Design of a 1000 Megawatt Electric Sodium-Cooled Fast Breeder Reactor," (ibid.) also contributes to the analysis of accidents which disrupt system reliability. Studies of smaller liquid metal reactors by General Electric contribute to the analysis of scale: cf. H. E. Dodge, et.al., Conceptual Design of a 565 Megawatt Electric Fast Ceramic Reactor (GEAP-4226, April 1963), and K. N. Horst, et.al., Core Design Study for a 500 Megawatt Fast Oxide Reactor (GEAP-3721, December 28, 1961).

life cycle from fabrication to removal from the core, do not differ in kind from those in the major designs of other types of reactors.[2]

To find the dimensions of these elaborate capital systems K and less elaborate fuel requirements F, the design studies have to contain three sets of specifications. They must indicate the minimum capital requirements for a given fuel inventory and given level of thermal capacity. There must be sufficient information, presumably from parametric studies, to find the dimensions for $\partial K / \partial F$ at some relevant level of capacity. Studies of different-sized reactors must be comparable, so that the effects of scale on the relative utilization of inputs can be assessed.

It is not apparent that these requirements can be met exactly from information in the four liquid metal fast reactor design studies. The four designs were undertaken on the basis of limiting ground rules. Both capacity and capital were given before the studies could begin (the second as a result of the first, and of the specification of required thermal efficiency), and the design fuel inventory was constrained by the further requirement that the

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[2] This is not to deny the existence of different configurations in the design of the fuel core. For example, there are striking differences among the four studies themselves, as shown in the view of the Reactor Engineering Division, Chicago Operations Office, U. S. Atomic Energy Commission, An Evaluation of Four-Design Studies of a 1000 Megawatt Electric Ceramic-Fueled Fast Breeder Reactor (COO-279, December 1, 1964), in the diagram of reactor core arrangements of the four design studies on the first page. But these core configurations are not specific to the liquid metal fast reactor; in fact, the configurations in the gas and steam cooled reactors discussed below are more similar to those of Combustion Engineering Corporation and General Electric than these two designs are to the Westinghouse and Allis-Chalmers designs for the liquid metal reactor.

study only consider maximum core outlet temperature.[3] The parametric studies do not point directly to $\partial K/\partial F$, although those on "specific power" (equal to MW_t/F) are relevant. Comparisons of input levels for different-sized reactors are limited to those from the General Electric major design for 2500 MW_t and an earlier study of a 1395 MW_t sodium fast reactor by the same company.

There are some indications of the required information, however, that allow a first approximation to β' and ψ'. The General Electric study, and that of Allis-Chalmers, indicate capital expenditures in great detail. If capital for the production of heat energy in the form of steam at 1000°F at 2500 p.s.i.a. consists of all systems both within the reactor and for the transfer of heat to the sodium and then to the steam generator, then these estimates show costs for relevant capital inputs. The General Electric expenditures on relevant equipment include those on (a) the reactor vessel and internals, (b) primary sodium pumps, drives, and piping, (c) intermediate heat exchangers, (d) secondary sodium pumps, drives, and piping, and (e) the steam generating system. They should not include payments for the turbine generator building, the generator unit and its accessory electrical equipment. The total of relevant expenditures, termed ΣPQ for component price

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[3] The implications of these constraints cannot be known with any exactitude. But it would seem quite likely that they stand in the way of finding the least cost combination K_1, F_1 for given MW_1 even with the factor prices stated in the AEC Guide to Nuclear Power Cost Evaluation. Both the constraints on capital and on fuel set the levels of utilization of these two inputs, rather than allowing the ratio of marginal products of the inputs to be equated to the ratio of factor prices. Of more concern is the possibility that the observation of F_1 is not the minimum amount required to produce MW_1 with any stated K_1 -- the specification of maximum core outlet temperature may require more F_1 than is necessary. Then variation of the K,F combination will not lead to an approximation of the least cost combination of inputs, because the observed combination may include too much of both inputs.

P multiplied by quantity Q, comes to $\$81.1 \cdot 10^6$ and the total number of components ΣQ is 466.[4] Not all components are of the same size, nor do they involve the same amount of fabrication and engineering of the same metals; weighting each by its price -- so that those with higher prices are assumed to be larger -- then the average size of a component is $\Sigma PQ/\Sigma P = 81.1(10^6)/44.4(10^6) = 1.82$ units of capital. Then total capital comes to $(\Sigma PQ/\Sigma P)\Sigma Q = (1.82)(466) = 848$ units.

The quantity of fuel required for 2500 MW_t is investigated in all four design studies. Total fuel mass is divided between "the core" and "the blanket" surrounding the center of fission; the latter has different qualities and functional use when compared to the former; for one, plutonium in the core fissions to produce heat energy and neutrons for converting fertile blanket uranium, which is in relative abundance, to more fissionable plutonium. Capacity to produce thermal megawatts is determined by the kilograms of fissile plutonium at any time during the lifetime of the capital equipment. The initial plutonium loading provides this capacity at first (since this amount is capable of producing 2500 MW_t). But a loading lasts two to three years, and capital is in operable condition for thirty, so that the inventory required to provide lifetime capacity is the sum of the loadings -- that is, the sum of the present discounted values of the volumes

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[4] Cf., the General Electric Company, Liquid Metal Fast Breeder Reactor Design Study, op.cit., Section 2.8, "Economics Data." The number of components is estimated from counting items of equipment in Tables 2.8.2.1 "Reactor Equipment Cost Summary," and in the plant layout charts and diagrams shown throughout the report.

loaded.[5] In the General Electric design, there are initially 2357 kilograms of fissile plutonium, and in the Allis-Chalmers design there are 2910 kilograms;[6] the first is consistent with the 848 units of capital, since it is from the same study, and is used to indicate a single observation of (K,F) for one MW_t. This volume of fuel is to be loaded every two to three years, and is to include newly-purchased plutonium or that bred in previous loadings. New fissile fuel from one source or the other, termed M as compared to the original loading M_o, is a fraction of M_o in any one year given by B^{-1/t_1} with B the ratio of fissionable atoms produced to those consumed in the core lifetime equal to t_1.[7] For the General Electric case, with a breeding ratio of

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[5] The time perspective for planning productive capacity is from the initial installation of equipment over the lifetime of that equipment. The planner should consider capital and fuel in the same dimension -- that quantity of each input factor required to provide 30-year capacity to produce a certain amount of thermal energy. The capital capacity is indicated by the equipment installed during initial construction, other than certain items of equipment which have to be replaced before the 30-year lifetime is complete. The fuel required is the sum of annual loadings of fuel and blanket uranium, terms ΣM_i for $i = 1$ to $i = 30$. But these annual loadings do not all take place at the time of the initial installation of the equipment; rather they can be postponed with consequent savings of investment capital shown by the annual rate of return r on such invested capital. Each loading M_i can be discounted by $(1 + r)^i$, or the required fuel inventory equals $\Sigma M_i(1+r)^{-i}$.

[6] The differences in fuel inventories can be accounted for in terms of temperature and pressure conditions in the core which utilize fuel more intensively in the first case, and less intensively in the second case. If the technologies assumed in the design studies are the same, then the two values of F are observations for fuel and capital in different combinations producing the same amount of output. That is, the design studies show F_1, F_2 for $MW_t = MW_{t*} = f(K,F)$, an isoquant in the capital and fuel dimensions of minimum amounts of capital and fuel.

[7] Cf., General Atomics Division of the General Dynamics Corporation, A Study of a Gas-Cooled Fast Breeder Reactor: Initial Study. Core Design Analysis and System Development Program, Final Summary Report (GA-5537, August 15, 1964), Section 12.1, "Fuel Cycle Cost Equation," pp. 163-167.

close to 1.2 and a core lifetime of 2.57, the required additional loading declines from the initial loading at a rate of 7.1 per cent per annum; also the present value of all future loadings declines at a rate of at least 4.75 per cent for each year further removed from the time at which the initial loading takes place, given that the minimum charge for fuel in use is 4.75 per cent per annum, so that loadings in year t are equivalent to the initial loading discounted by 11.85 per cent for each intervening year. The sum of loadings over the reactor lifetime is approximately $M = (2357)/(2.57) \sum_{1}^{30} (1.1185)^{-t}$, or 7045 kilograms. Then $MW_t = 2500$, $K = 848$ and $F = 7045$ is an approximate single observation of capacity and capacity input requirements for the general LMFBR production function.

A tradeoff of fuel for capital is involved in a change in a basic construct of the design: increasing the "specific power" of the system, as measured in energy kilowatts per kilogram of fissile plutonium,[8] raises the capital and reduces the fuel requirements. Higher specific power can be realized by increasing the thermal conductivity of the fuel (from adding beryllium oxide to the fuel rods, for example) or by raising fuel and coolant temperatures; both add to the thermal stress or corrosion of the fuel assembly included in capital equipment, and add to capital items in the shielding and heat transfer systems, and both reduce the fuel inventory. Additions to capital from increasing specific power are shown in the Combustion Engineering study as a substantial increase in the number of fuel

[8] The "specific power" of a reactor design is defined as thermal megawatts of capacity divided by initial or equilibrium fissile fuel inventory. In economic terms, specific power is the average productivity of fuel MW_t/F.

assembly components, and a decrease in the radius and length of each component; this results in increased fabrication of the rods and of the smaller fuel pellets -- and, as a first approximation, the increased fabrication is equivalent to a 20 per cent increase in components for a 100 per cent increase in specific power. The increased corrosion and deterioration due both to increased temperatures and accompanying increased pressures require quality improvements and more frequent replacement of components in reactor vessels and their internals -- again, in the Combustion Engineering study, equivalent to a 20 per cent increase in these components given a 100 per cent increase in specific power. The reduction in fuel inventory from increased specific power is shown in both the General Electric and Allis-Chalmers studies. By "doubling the core power density and specific power, simultaneously reducing the number of batches to keep the same refueling schedule -- [and accepting] a penalty in the physics area ..." the input plutonium content is decreased, although the content per batch is increased 1.3 per cent, and the breeding ratio is reduced by approximately 0.07; the net effect in the General Electric design is that fuel inventory is reduced approximately 10 per cent.[9]

The tradeoff of fuel for capital $\partial K/\partial F$ in the range of design specific powers can be calculated from these indicators. If the increase in fuel assemblies and reactor internals as shown by Combustion Engineering is applied to the number of units of capital shown in the General Electric design, then 33 more units of capital are required to increase power by 100 per cent. This reduces fuel by 705 units, so that $\partial K/\partial F \sim -5/100$, or 5 units of capital are added for each 100 units of reduction of fuel.

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[9] Cf., the General Electric Company, Liquid Metal Fast Breeder Reactor Design Study, op.cit., (GEAP-4418), pp. 5-7.

The requirements for capital and fuel at different scales of thermal energy output have not been investigated as part of any one design study. General statements have been made on the advantages of large scale: The AEC design handbook is based on output increasing at a faster rate than inputs, so that larger plants have lower capital/output and fuel/output ratios than smaller plants; but the Westinghouse design study, in considering both small and large versions of a single liquid metal reactor, shows little decrease in inputs per megawatt capacity at large scale.[10] The only detailed studies of different sizes of the same reactor have been two General Electric designs, that for the 2500 MW_t liquid metal fast reactor and that for a 1395 MW_t reactor based on the same coolant cycles and core geometry. The conceptual design for the first indicates, in the physical units used here, K = 848 and F = 7045; the second design provides most, but not all, necessary information for constructing comparable estimates of K and F for MW_t = 1395.

The 1395 MW_t design is based on the same technology -- the study was completed in 1963, rather than 1964, but it has the same flow diagram as the larger reactor.[11] Four primary sodium pumps move the liquid coolant through the bottom and then out of the top of the reactor core to four intermediate heat exchangers which also contain sodium; the secondary loops then carry the heat energy to a once-through steam generator or to steam reheater loops. Such

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[10] Kaiser Engineers, Guide to Nuclear Power Cost Evaluation, Production Costs (TID 7025, Vol. 5, March 15, 1962), Section 306. For the Westinghouse design study, cf. Westinghouse Electric, Liquid Metal Fast Breeder Reactor Design Study, (WCAP-3251-1, January 1964).

[11] Compare Figure 2.4 in GEAP 4418, the 2500 MW_t design study, with Figure 9 in GEAP 4226, the basic 1395 MW_t design study. The flow diagrams are the same, at least in these simplified versions of the two systems.

a system differs only in the number and size of loops or other components, when compared to the 2500 MW_t General Electric liquid metal design, but not the techniques of heat transfer, the use of an intermediate transfer network, or the design of the reactor vessel and internals.[12] Sodium is transported out of the reactor at 1050°F and 26 p.s.i. at the rate of 60 x 10^6 lb./hour in the smaller reactor, and at the same temperature and pressure -- but in larger quantity, at 86 x 10^6 lb./hour -- in the larger reactor. The "product," evidenced as heat energy transported by the sodium to the pass-through steam generator, would seem to be the same.

The number of units of capital are $(\Sigma P^*Q^*/\Sigma P^*)\Sigma Q^*$ for the small reactor, for comparability with capital $(\Sigma PQ/\Sigma P)\Sigma Q$ in the larger system described above. Capital expenditures for the 1395 megawatts capacity are projected as $89.06 million, of which $62.9 involve components and the construction of the system ΣP^*Q^* comparable to the $81.1 million for the 2500 MW_t reactor. The number of components ΣQ^*, and the sum of prices ΣP^*, are not shown in the study, but a sample of prices for reactor equipment is identical to the sample in the design for the larger reactor,[13] so that $\Sigma P^* \sim \Sigma P$. Given the similarity in the designs of the two systems, the number of components should not be greatly different. Then both $\Sigma P^* \sim \Sigma P$ and $\Sigma Q^* \sim \Sigma Q$. The difference between systems is in the size of components -- the weighted average size of those in the smaller reactor $\Sigma P^*Q^*/\Sigma P^*$ should be smaller. Then units of capital $(\Sigma P^*Q^*/\Sigma P^*)\Sigma Q^*$ equal $(\Sigma P^*Q^*/\Sigma P)(\Sigma Q)$, which is $[(62.9)/(44.4)](466)$ or 660 units.

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[12] That is, the basis for differences in rates of flow, and in the size and number of components at any stage in the heat transfer system, is in requirements for larger heat transfer by faster flow of sodium in the larger reactor.

[13] This is the case at least for those items, such as steel plate and croloy, for which price comparisons are possible. Cf., GEAP-4418, Table 2.8.2.1 and GEAP-4226, Table IV.

The smaller reactor uses more fuel for producing heat energy, in the sense that the initial core loading of fissionable plutonium, equal to 1200 kilograms, is larger per megawatt of capacity than that in the 2500 MW_t design reactor. This smaller unit does not "breed" as well, because capture of neutrons by fertile uranium is less complete in a smaller core; the ratio of bred to burned plutonium is 1.08, rather than 1.2. With less fuel produced in the core, the additional loadings occurring every three years are larger relative to the first core inventory. The core inventory after the first equals $M_1 = M_0 B^{-1/t_1} = 1200(1.08)^{-1/3.07}$ which is equivalent to an annual discount of 3 per cent of original inventory, rather than of 7.1 per cent in the larger design. Total lifetime inventory, viewed from the period of plant construction, is $1200/(3.07) \sum_{1}^{30} (1.0775)^{-t}$ equal to 4270 kilograms.

The values of K and F for MW_t in the two reactors, and of $\partial K/\partial F$ for the larger reactor, provide a first approximation to the expected LMFBR production function. The two designs indicate, for the equation $[\beta'(\log K_1/K_2) + \psi'(\log F_1/F_2) = \log(MW_1/MW_2)]$, that $\beta'(1089) + \psi'(2475) = 2534$. The tradeoff of capital for fuel on specific power $\partial K/\partial F = -\psi' K_1/\beta' F_1 = -5/100$ or, for the design value of 2500 MW_t, $\psi'(848)/\beta'(7045) = 5/100$. Solving these two equations yields $\beta' = 1.19$ and $\psi' = 0.49$ or a description of the function as $MW_t = K^{1.19} F^{0.49}$.

Even the first approximation provides a characterization of the liquid metal breeder. The extensive heat adsorption of liquid sodium adds to the productivity of capital, but the equipment required for reliability of the heat transfer system at any scale is about the same as at any other scale of operations. This aspect of technology has an interesting effect: it takes

small additions to capital to make large additions to thermal energy capacity. It is shown by the values of the marginal product of capital in the ranges of F, K in the two design studies; this marginal product, or $\partial MW_t/\partial K = \beta MW_t/K = 1.19\ MW_t/K$ increases over the range from 1395 MW_t to 2500 MW_t. If capital is added to a given level of fuel, the addition increases output by a greater amount up to the level of K implying the maximum temperature of heat energy production and transfer within the available technology.

A second approximation to the liquid metal production function has to account for deviations of output from that for which the reactor is designed. There is a probability distribution of outputs from spaced runs with a given amount of equipment and fuel. Certain combinations $MW_i = f(K_i, F_i)$ have greater probability of zero MW_t: driving the core at limiting thermal conditions increases distortion in the fuel rods and the risk of melting the fuel when there is a random further increase in temperature; it also reduces the margin between maximum permissable reactivity and that during transient changes in power, which involves the risk of excess reactivity rendering fission uncontrollable.

The changes in reactivity with respect to temperature and power determine the stability of the system. Temperature induced reactivity changes $\partial\rho/\partial T$, with ρ defined as the per cent deviation from steady state levels of fission activity, are random events because of the random nature of temperature changes; if a temperature increase is not to increase fission, so as to increase temperature and fission once again and render the system unstable, then values of $\partial\rho/\partial T < 0$ are required. In many fast sodium-cooled reactors, increases in temperature reduce fuel and sodium density, and reduced density

implies $\partial \rho / \partial T < 0$. But in reactors with a compact core of minimum possible size fissioning at high temperature, the fast neutron energy spectrum may cause $\partial \rho / \partial T$ to have the opposite sign. That is, it is possible that (1) $\partial \rho_1 / \partial T < 0$, where ρ_1 results from reduced sodium density, but the magnitude of this negative effect decreases as the compactness of any core increases; (2) $\partial \rho_2 / \partial T > 0$ where ρ_2 is the effect of sodium in "degrading the neutron energy spectrum" so as to increase fission from the available neutrons; (3) the size of $\partial \rho_2 / \partial T$ becomes larger with larger reactor size. Thus for larger reactors with compact cores, the sum total effect may be either $\partial (\rho_1 + \rho_2) / \partial T < 0$ or $\partial (\rho_1 + \rho_2) / \partial T > 0$. The reactors with relatively more compact cores operating at higher temperatures are more prone to the latter. These reactors have to be shut down when there are large temperature transients, to prevent permanent core melting and other "forced outage" for long periods of time.

The three design studies of the large liquid metal reactor have been concerned with forced outage and thus have become involved with a tradeoff of reactivity decreases for higher costs of operation at design levels of output. Decreasing the volume of fuel in the core, and increasing the capacity of the sodium coolant and transfer systems so as to maintain sodium outlet temperatures, results in the undesirable larger positive values of $\partial \rho / \partial T$.[14] But this design variant also increases the rate of production of new fissile

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[14] Cf., The General Electric study, Liquid Metal Fast Breeder Reactor Design Study, op.cit., Section 3.3.2, "Evaluation Techniques for Survey Data and Their Graphic Representation, Survey of Fast Reactor Cores"; cf., also K. P. Cohen, and G. L. O'Neill, op.cit. Section 3.2, "Reactor Design."

fuel from the original fuel inventory -- less neutrons are "lost" in the compact core, and more are "captured" by U(238) to form Pu(239). There is a gain in expected design productivity, then, from reducing the certainty of this design productivity. The General Electric tradeoff appears to be an addition to reactivity $\rho = .006$, with an undefined increase in the probability of downtime from an overheated reactor, for every one per cent increase in design temperature. The higher temperature increases the breeding ratio -- in this design by as much as 20 per cent -- and thus reduces the fuel inventory requirements.

The second approximation accounts for departures of capacity from the design values as a result of transient temperature changes, and for the magnitude of the departures as a function of the design temperature and capital/fuel ratio. As an experiment in such accounting, the factor U in $MW_t = K^{\beta'}F^{\psi'}U$ can take the form $U = e^{-x}$, where x is the product of the temperature variant $(T - T^*)$ times the capital/fuel ratio K/F. This description of U shows the expected effects on the reliability of thermal energy capacity from operating at the limits of temperature and pressure. When actual temperature T equals design temperature T^*, $U = 1$ and $MW_t = K^{\beta'}F^{\psi'}$, the design values of inputs and capacity output. But when a transient results in $T = 1.05T^*$ at $T^* = 1000^{\circ}F$, and K/F is as low as 1/5, then $U \sim 0$ and available capacity approaches zero as well. A 5 per cent deviation in temperature closes down the reactor, either by a melting accident or to prevent such an accident. This may not be the case in a smaller reactor, however; with $T = 1.05T^*$, with $T^* = 600$ and $K/F = 1.10$ then $U > 0$ and capacity is greater than zero, although not very much greater. Also, with transients of the order of $T = 1.005T^*$ then actual

capacity may still be more than 90% of design capacity. Temperature deviations in this reactor are expected to result in penalties of this magnitude on available capacity; at least the safety rod and fast shut-down systems in the General Electric design can be expected to go into effect for changes in temperatures of this magnitude.[15] Then this approximation of the production function is

$$MW_t = K^{1.19}F^{0.49}e^{-(T-T^*)(K/F)}$$

for instantaneous capacity MW_t, capital K, fuel inventory F, and coolant outlet temperature T.

The Steam Cooled Fast Breeder Reactor

The uranium-plutonium rods making up the core of any fast reactor can be put together to transfer heat energy directly to steam, rather than to an intermediate sodium loop for re-transfer to steam. The design of the core would not differ greatly from those in the sodium reactor studies; oxides of the two fuels in five-foot rods can be arranged in hexagonal clusters to allow the pass-through of pre-heated steam in the same manner as the pass-through of liquid sodium discussed above. But the design of the heat energy transport system greatly departs from that for the liquid sodium reactor.

Descriptions of flow from the water source to steam entering the reactor, and then from the reactor to the turbine generators, indicate the

[15] Ibid.

great simplicity of the direct steam system. Steam condensers, both in series and in parallel with the turbine generator, send a mixture of water and steam first through low pressure and then high pressure feed pumps to pass-through steam heaters. Steam emerges from the high pressure heaters at approximately 500°F and, in reactors of 1200 MW_t or more, at the rate of more than 2.5×10^6 lbs./hour. A drum-type or Loeffler boiler then mixes this steam with hotter steam from the reactor and transmits it to a circulator at close to 600°F and at the pressure of approximately 1200-1500 p.s.i.a. By the time that the discharge from the circulator reaches the reactor, it is at more than 600°F and 1500 p.s.i.a.; it enters the reactor from below and exits from the top at 950°F and about the same pressure. This steam is then passed directly through the turbine generator to produce electricity; there are no intermediate loops or other kinds of heat transfer surfaces, but rather the reactor coolant is used to turn the blades of the turbine.

The effects of simplicity are found in the expenditures on capital equipment to construct and operate the proposed steam cooled breeder reactor (SCBR). Capital costs are lower than for the liquid sodium reactor of equal capacity, as a result of the elimination of primary and secondary heat transfer systems, but also as a result of reductions in shielding and back-up equipment to guarantee the reliability of water-volatile sodium. This is to suggest that, for the SCBR production function $MW_t = K^{\beta'} F^{\psi'} U(K,F)$, the value of K_1 for given MW_1 and F_1 is probably smaller than for the LMFBR. The value of F_1, or the inventory of fissile plutonium in the core, is another matter; steam is not as good a heat conductor as liquid sodium, nor are the energy-flux conditions in a steam environment as conducive to breeding, so that more fuel is needed for the SCBR than for liquid sodium coolant systems.

The design studies of the 870 MW_t steam cooled breeder provide a single observation of capacity, capital, and fuel to show these effects. Comparison with a design for a 2500 MW_t steam system -- which is in bare outline form -- allows some inference on economies of scale and ultimately some basis for assessing differences from the liquid metal reactor.

Most of the studies of the smaller SCBR were done in the late 1950's and early 1960's, by the United Nuclear Corporation, on the basis of technology then available or expected to be available by the early 1970's.[16] The early dating of this technology does not make it non-comparable with that of other reactor types, because the technology of this system is well developed at the present time. The direct cycle of steam through the reactor to the turbine is based upon extensive experience with this system in light water thermal reactors. Developmental problems lie ahead in fuel rod exposure to high temperatures and long periods in core inventory, since there has been little experience with prolonged burnup of fuels in a fast reactor; but these problems arise for the liquid metal and gas reactors and the solutions assumed in the SCBR studies are not different from those assumed elsewhere.

The "present-day" designs call for capacity to produce 870 megawatts of thermal energy for steam turbine generators that are to operate at 36.4

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[16] Cf., G. Sofer, R. Hankel, L. Goldstein, G. Birman, Conceptual Design and Economic Evaluation of a Steam Cooled Fast Breeder Reactor (United Nuclear Corporation, Development Division, White Plains, New York: work performed under Contract AT(30-1)-2303(XII) with United States Atomic Energy Commission, November 15, 1961). This study was based in part on the analysis of capital costs in Kaiser Engineers Division, Henry J. Kaiser Company, Steam Cooled Power Reactor Evaluation Capital and Power Generation Costs (TID-12747, Oakland, California, March 17, 1961). Cf., also, G. Sofer, et.al., Steam Cooled Power Reactor Evaluation, Steam Cooled Fast Breeder Reactor, (Nuclear Development Corporation, NDA-2148-4, April 15, 1961) for preliminary studies of the contrasts between near-term plants and 1975 reactors.

per cent net efficiency, and the 1975 designs call for 825 thermal megawatts to produce the same amount of electricity output. The thermal capacity provides steam at 945°F and 1400 p.s.i.a. in the 1960's, and 1060°F and 2000 p.s.i.a. by 1975. By the middle 1970's the "product" of heat energy is approximately the same as that from the liquid metal fast breeder reactor.

Capital expenditures ΣP^*Q^* to provide the capacity are $\$35.2(10^6)$ for construction in 1967; the total of prices for components ΣP^* is $\$17.1(10^6)$ and the total number of components ΣQ^* is 59. For comparability with other reactor systems, three calculations must be made. First, the price-weighted average number of units $(\Sigma P^*Q^*/\Sigma P^*)\Sigma Q^*$ is (35.2/17.1)(59) or 121. Second, this amount of capital has to be "scaled" so that a single item is of the same size as one in the liquid metal fast breeder. If an item requires the same amount of engineering and fabrication per square foot in all systems, then any difference in expense among systems is due to difference in size. To count units all of the same size, multiply that unit costing three times as much by three. The amount of capital in the steam cooled breeder is based upon an average price of $\Sigma P^*/\Sigma Q^* = \$0.30(10^6)$, while the unit of capital in the LMFBR was priced at $\$.09(10^6)$; since capital in the first reactor is relatively larger in price terms, then the 121 units increase by (.30/.09) to 403 units comparable to those in the LMFBR. Third, costs for constructing this reactor in 1975 are estimated by the United Nuclear Corporation designers to be 8 per cent less than these costs in 1967; if costs are reduced because of technical improvements in components, but not because of reduced prices,[17] then the

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[17] The assumption is supported by G. Sofer, et.al., Conceptual Design in Economic Evaluation of a Steam Cooled Fast Breeder Reactor (op.cit., NDA 2148-5), p. 117, Sections 1, 2, 3.

number of components is reduced from 403 to 371 during the time period for constructing both the SCBR and the LMFBR. It is estimated that K = 371 is the required amount of capital in this design of a steam cooled breeder reactor with capacity of 825 thermal megawatts.

The inventory of fuel to provide such capacity begins with a loading of 1370 kilograms of fissile plutonium and with approximately seven times this much fertile uranium in the core and surrounding blanket for breeding more plutonium. But breeding is limited in this design because the intensity of neutron radiation is reduced by the steam coolant and because the high working temperatures and pressures require fuel rod claddings which reduce neutron absorption by the fertile uranium.[18] Also, the coolant reduces the active lifetime of fuel in the core; burnup is expected to be limited to 60,000 MW-days per metric ton of core fuel, so that the number of days for which the core is in use adds up only to 2.3 years. The experience both with breeding and burnup affects the inventory required of fuel over the thirty-year lifetime of the reactor. With the breeding ratio B equal to 1.2 and t_1 equal to 2.3, the annual rate of reduction of inventory B^{-1/t_1} from breeding is 7.5 per cent. Carrying changes of 4.75 per cent reduce the present value of future loadings by that amount as well. The inventory, then, is the accumulation of an initial annual inventory of (1370/2.3) discounted by 12.25 per cent for each year, or $(1370/2.3) \sum_{1}^{30} (1.1225)^{-t} = 5080$ kilograms of fuel.

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[18] Cf., R. A. Mueller, F. Hofman, E. Kiefhaber, and D. Smidt, "Design and Evaluation of a Steam Cooled Fast Breeder Reactor of 1000 MW Electric" (Kernforschungszentrum, Karlsruhe, Institute Für Reaktorentwicklung PSB-Berict Nr. 119/66), p. 3.

The estimate of lifetime fissile fuel requirements completes this observation: for 825 MW_t of instantaneous capacity in a 1975 steam cooled breeder reactor, K = 371 and F = 5080 over the equipment lifetime of thirty years.

The tradeoff of capital for fuel $\partial K/\partial F$, as implied by changes in specific power, is not shown in the design studies of the 825 MW_t reactor. But there are indications of the effects from increasing specific power in the 1975 design over that for the 1967 design. The "near term reactor" has specific power of 505 kilowatts per kilogram of fissile plutonium, while the "1975 reactor" has specific power of 577 KW/Kg; if the effects on K and F of other differences between the two reactors can be removed, then the residual effect must be due to the variation in the specific power.

The first reason for differences is that the 1975 reactor benefits from technical progress which reduces K anf F, while the near term reactor does not. The fruits of progress include a substantial reduction in K: "The use of vapor suppression type of containment and ... simplification in starting, shutdown, refueling, and fuel handling equipment"[19] are estimated to reduce expenditures ΣP^*Q^* by much more than 8 per cent. The net effect of higher specific power, which increases capital requirements, and technical progress which reduces these requirements, is an 8 per cent reduction in expenditures. Reductions in capital from technical progress alone should exceed 2 per cent per annum -- because of a modest annual increase in productivity of capital following from "learning by doing" with the direct cycle in the light water thermal reactors installed from 1967 to 1975. The elimination

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[19] G. Sofer, et.al., op.cit., p. 117.

of 16 per cent of actual capital, so that K of 403 in 1967 terms is equal to 338 in 1975 terms, can be attributed to technical progress separate and independent from attributions for changes in specific power.

The second possible reason for differences in K is in the scale of the 1967 and 1975 reactors. Total thermal capacity in the near-term plant is 876 and in the later plant is 825 so that there would appear to be larger scale in the first plant. But the difference is a result of increased thermal efficiency from the higher temperatures and pressures which cause the higher specific power in the second plant -- that is, a result of producing a smaller amount of steam of higher quality which can in turn produce the same amount of electricity -- so that the "outputs" of the two plants are the same. Any alleged difference for scale of operations cannot be taken into account.

For 1975 technical conditions as the standard of reference, then, specific power is 577 and K = 371, or specific power is 505 and K = 338. Both observations assume technical progress, but the first is the value for specific power and capital estimated for the 1975 design, while the second is the design value for the 1967 reactor, with assumed 1975 technical progress, but at the lower 1967 specific power.

Increasing specific power reduces the inventory of fuel required for any desired level of thermal capacity. This is more than a matter of defining "higher specific power" as "greater output capacity per unit of fuel"; it is a variation of design characteristics to attain steam temperatures and pressures, and also those for fuel rod cladding, that approach the limits allowed by technology. These temperatures are conducive to more complete consumption of fuel, and thus to a longer life for any core

inventory; but higher temperatures reduce breeding because they require fuel rod cladding materials that absorb neutrons. In the United Nuclear designs, the prolonged burnup at higher temperature is the dominant effect. The burnup of a core inventory under the lower specific power conditions takes 33,000 MW days per ton of fuel on the average, when the lifetime inventory is approximately 5703 kilograms (given a 2 per cent annual rate of technical progress in fuel use as in capital use for each year before 1975). The burnup increases at the higher specific power to 60,000 MW-days, while the breeding ratio is reduced only by .02, so that the present value of total lifetime fissile fuel inventory is reduced from 5703 to 5080 kilograms.

Increasing specific power has both capital and fuel effects on the relevant designs, then. Capital is increased by 33 units, and fuel is decreased by 5703 to 5080 or by 623 units, so that $\partial K/\partial F \sim +33/-623$ or $-5/100$.

The 825 MW_t reactor is reasonably well defined, both in terms of one observation of K,F for MW_t and of variations in specific power indicating other combinations of possible K,F for this level of capacity. The 2500 MW_t steam reactor has not been defined in detail, however, by revising the United Nuclear designs to attain this level of capacity or by independent design. One consequence is that there are no studies of the effects of scale in the range of 1200 MW_t to 2500 MW_t comparable to that for the liquid sodium reactor. There are only limited indications of economies of scale from outlines of designs for the larger reactor.

One study by Kernforschungszentrum Karlsruhe shows the outlines of a design for a 2500 MW_t steam cooled reactor but only points to the need for an evaluation of the tradeoffs between breeding, fuel burnup, and core reliability. There is some indication that the tradeoffs should result in optimal capital that costs close to $\$119.0(10^6)$, and in an initial fuel inventory of 2232 kilograms of plutonium, where capacity is 2517 megawatts of thermal energy.[20] Two assumptions can lead to estimates of units of capital for 2517 MW_t from the first statistic. First, if the expenditures on turbine generator components and structures are equal to the $\$43.9(10^6)$ shown in the 2500 MW_t liquid sodium design reactor,[21] then expenditures on the heat energy source $\Sigma P'Q'$ are $\$119.0(10^6) - \$43.9(10^6) = \$75.1(10^6)$. Second, if these expenditures $\Sigma P'Q'$ are for the same number of components at the same charge per square foot of fabrication and engineering as in the 825 MW_t steam cooled reactor, then units of capital $(\Sigma P'Q'/\Sigma P')\Sigma Q' = (\Sigma P'Q'/\Sigma P^*)\Sigma Q^*$ where P^*, Q^* are those shown in the United Nuclear design studies of the smaller steam cooled fast breeder. This assumption should be modified: the Karlsruhe design shows six independent coolant circuits -- each with a Loeffler boiler as the last link before running steam into the reactor core -- rather than the two circuits shown in the United Nuclear designs, so that the number of units of capital in the larger design $\Sigma Q' = \Sigma Q^* + 26 = 85$, where the additional

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[20] R. A. Mueller, et.al., op.cit.

[21] This is not an unrealistic assumption, given that the temperature and pressure conditions for steam are approximate to those in the liquid sodium fast breeder reactor.

26 units result from a three-fold increase in circuitry. But the prices of all units of a given size are the same in the two reactors, so that units of capital are $(\Sigma P'Q'/\Sigma P^*)\Sigma Q' = [75.1(10^6)/17.1(10^6)]85 = 375$. In units of capital comparable in size to those for the liquid metal fast breeder reactor, for post-1975 technological conditions, 375 increases to 833.

The second statistic in the Karlsruhe study -- the initial fuel inventory -- is not the only indicator of the behavior of the fuel cycle. The proposed design is to operate at the technical limits of temperature and pressure, so as to guarantee burnup of core fuel of between 50,000 and 100,000 MW_t-days per ton. This prolongation of exposure in the core increases the lifetime of the fuel elements to approximately 3.5 years but greatly reduces breeding; as shown here, the breeding ratio falls from more than 1.2 to close to 1.1 kilograms of fissionable fuel produced in the core and blanket for every kilogram consumed. Then the rate of discount $(B)^{-1/t_1} = e^{-.027}$ and the inventory over the plant lifetime of thirty years is $(3323/3.5)\sum_{1}^{30}(1.075)^{-t}$ equal to 11,125 kilograms of plutonium.

These values of K and F can be used along with the detailed observation from the United Nuclear designs to show the extent of economies of scale. The Karlsruhe design indicates $K' = 833$, $F' = 11,125$, and $MW_t' = 2517$, at specific power of 720 kilowatts/kilogram of plutonium; these can be considered equal to $K' = 693$, $F' = 13,900$ and $MW_t' = 2517$ at the specific power of 577 KW/Kg of the smaller United Nuclear design reactor. The smaller reactor indicates $K^* = 371$, $F^* = 5080$, and $MW_t^* = 825$. In the general form of the production function $MW_t = K^{\beta'}F^{\psi'}$, the two observations show the coefficients of β and ψ as $\log(2517/825) = \beta'\log(693/371) + \psi'\log(13,900/5080)$, or

$4844 = 2713\ \beta' + 4371\ \psi'$. The estimate is a first indication that economies of scale are limited: the sum of β' and ψ' is not much greater than one, so that a doubling of capital and fuel leads to little more than a doubling of capacity. A second indication, leading to separate estimates of β' and ψ', confirms the first. The equation $4844 = 2713\ \beta' + 4371\ \psi'$ can be put together with estimates of $\partial K/\partial F$ to provide two equations with two unknowns, β' and ψ', that can be solved for each unknown. The second equation is $\partial K/\partial F = \beta\psi' K/\beta' F = -5/100$, or $\psi'(371)(10^2) = \beta'(5)(5080)$ with $K = 371$ and $F = 5080$, and the two equations solve for $\psi' = 0.58$ and $\beta' = 0.85$. The first approximation to the steam cooled fast reactor production function can be described as

$$MW_t = K^{0.85} F^{0.58},$$

and the description again points to relatively limited economies of scale, since the sum of the two exponents is less than the value of 1.68 observed for the liquid metal fast breeder reactor.

The Gas Cooled Fast Breeder Reactor

The direct steam cycle for transferring heat from the reactor is only one type of gas cooling system. Steam is a gas having acceptable heat transfer properties; helium, carbon dioxide, and sulphur dioxide are other gases that also can be used between the reactor and steam generators. There are "best routings" to be followed from fission energy to electric energy for each of these gases, and each such design differs somewhat from the others. Perhaps, however, the helium gas reactor can serve as the representative design, both because it is well advanced towards adoption in a sophisticated converter reactor demonstration plant and because it is distinctive with respect to a steam-cooled reactor.

Helium is introduced into the system outside of the reactor through filters and compressors, and it enters the reactor by means of large gas compressors at 500°F and 1050-1060 p.s.i.a.[22] The gas is blown up through the fuel core, where its temperature is increased from fission to 1150°F and its pressure is reduced approximately to 1015-1020 p.s.i.a., and then the gas is blown down through four steam generators surrounding the core. The blowers, core, and generators are all contained in a single vessel of pre-stressed concrete or steel and this vessel is contained within a secondary reinforced concrete structure. The helium emerges from the two pressure vessels at 500°F and 1000 p.s.i.a., for purification and storage, while steam emerges at 950° - 1000°F and 2400 p.s.i.a. for transfer to the turbine-generators. The circuit is completed -- outside of the pressure vessels -- by condensing the steam and running the condensate through boiler feed pump trubines to the steam generator intake into the pressure vessels, and by taking the helium from storage to the gas compressors for entry into the pressure vessels.

This design shows strong differences from the steam-cooled reactor layout. The largest components -- gas compressors, fuel core, steam generators, primary and secondary pressure vessels -- include three that are not closely duplicated in the steam cooled reactor. The compressors in a 2500 MW_t gas reactor are so much larger than the pumps in the steam system that the technology extends beyond the steam experience -- and beyond present experience with all large pumps, for that matter. The steam generators serve as a heat transfer mechanism that is not duplicated in the steam-cooled reactor because that reactor operates on a direct cycle of steam from the core to the tur-

[22] These values are "representative" design values derived from the two most detailed design studies of a gas cooled fast breeder reactor: Oak Ridge National Laboratory, Reactor Division, Gas Cooled Fast Reactor Concepts (ORNL-3642, September 1964); and General Atomics Division, the General Dynamics Corporation, A Study of a Gas Cooled Fast Breeder Reactor, Initial Study, Core Design Analysis and System Development Program, Final Summary Report (GA-5537, August 15, 1964).

bine. Moreover, the primary and secondary pressure vessels are much larger and more complex because of the extra emergency requirements for containment of fission products in high-pressure helium when an accident opens the gas piping or the heat-transfer surface.

The design does not differ greatly from those for the liquid-metal fast breeders. It is simpler: the four gas-to-steam heat transfer loops in the primary vessel are once-through steam generators, not primary to intermediate loops carrying more coolant to final loops containing steam. System reliability is sought through particular design variants not found in the liquid sodium reactor. The helium reactor is designed to pour steam and water into the core if the coolant is voided by accident, so as to prevent core meltdown; but the liquid metal reactor cannot rely on this technique, because of the volatility of sodium in water, so that elaborate additions to capital are made to reduce the probability of voiding and elaborate auxiliary safety rod or sodium storage systems are constructed to cut off reactivity increases if voiding ever does take place.[23] As a result, the safety techniques and routings in the helium design are simpler. But the containment of the compressors, core and heat transfer surfaces in the pressure vessel in the helium reactor is substantially larger. It is prompted by the high pressures under which the helium operates, as compared to the ambient pressure of liquid sodium, and it adds to the capital components over and above those for containment in the liquid sodium reactor.

In terms of economic performance, where such performance is measured by $MW_t = K^{\beta'} F^{\psi'}$, the helium reactor has characteristics in contrast with both

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[23] Cf. K. P. Cohen, and G. L. O'Neill, "Safety and Economic Characteristics of a 1000 Megawatt Electric Fast Sodium Cooled Reactor Design," op.cit., and General Atomics Division, The Study of a Gas Cooled Fast Breeder Reactor, op.cit.

of the two types of reactors that reflect these design differences. The simplicity of the system for transferring heat energy is in contrast with the double-proof system using sodium and water, so that the number of units of capital is less for that reason. But the pressure in a gas transfer system exceeds that in either steam or sodium, so that the sizes of units of capital, particularly in containment, are increased. Fuel utilization is more complete with the gas coolant, since helium does not reduce the energy flux as much as steam, so that the breeding capability of neutrons from fission is enhanced. On the other hand, the gas is a relatively poor heat transfer medium. The thermal energy capability of a kilogram of plutonium -- that is, the specific power of the fissile fuel -- is lower than for either steam or sodium reactors at the same turbine throttle temperature and pressure, and this requires an increase in the plutonium content in a fuel loading for any given level of thermal megawatts. The net effects of these design characteristics on the relative requirements of capital and fuel for MW_t can be shown by defining β' and ψ' for a 1980 helium cooled fast breeder reactor.

The calculation of approximate β' and ψ' cannot be completed on the basis of design values from a single study, or even from a group of related studies similar to those done by General Electric on the liquid metal fast breeder. Detailed core analyses for gas-cooled reactors have been completed by the General Atomic Division of General Dynamics Corporation which provide the parameters for estimating fuel inventories for a wide variety of designs.[24] The study of Gas Cooled Fast Reactor Concepts by Oak Ridge National Laboratory

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[24] General Atomic Division, A Study of a Gas Cooled Fast Breeder Reactor, op.cit.

specifies the capital components to go with fuels and different gas coolants for different reactor capabilities.[25] A first approximation can be made to a single design from the characteristics for a helium-cooled 2600 MW_t fast reactor. Then this design can be varied for size and for specific power. First estimates of K_0, F_0, MW_0, and then of the effects on K, F, and MW_t of scale and of $\partial K/\partial F$ follow from this single design and its variants.

The most important design feature is the integrated primary and secondary containment vessel, with the gas compressors, fuel core, and gas-to-steam heat transfer surfaces inside the primary vessel. The ORNL study shows the costs of these components as more than $\$37.9(10^6)$ for capability of 2600 MW_t. With the addition of helium transfer, purification, oil removal and low pressure leak-off recovery systems all outside of the containment vessels, total capital costs are $\$65.0(10^6)$ for 86 components. The average size of a unit of capital $\Sigma PQ/\Sigma P$ equals $\$65.0(10^6)/\$33.2(10^6)$ or 1.96, so that the total amount of capital is (1.96)(86) or 169 units of roughly equal size. For size comparable to that of a unit in the liquid metal reactor, this is 723 units, because the average price per unit is 4.28 times that in the liquid metal reactor.

The inventory of fuel required to complement these units of capital can be shown only after specifying the uranium-plutonium oxide mixture out of those analyzed in the ORNL study and, for that mixture, after designating the specific power and thermal megawatts of capacity in terms used for fuel cycle analyses in the General Atomics study. Oxides of plutonium are

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[25] The Oak Ridge National Laboratory, Gas Cooled Fast Reactor Concepts, op.cit.

fabricated to perform at specific powers close to 800 thermal kilowatts per kilogram of this fuel; at least the fuel requirements are not reduced by higher specific powers, for the metal and fabrication inputs in the fuel rods increase at such a rate as to compensate for any further decrease in oxide content beyond 800 KW/Kg.[26] The initial inventory for 782 KW/Kg is shown as F = 3350. The lifetime inventory is equal to $(F/t_1) \sum_{1}^{30} (1 + d)^{-t}$ = 6700 kilograms, given that the annual discount "d" is 17 per cent because of a breeding ratio of 1.5, a core lifetime t_1 of 3.3 years, and an annual use charge of 4.75 per cent for plutonium. If there is no reduction in this inventory from increasing specific power to the more than 800 KW/Kg assumed in the study of capital from the ORNL design, then K_0 = 723, F_0 = 6700, and MW_0 = 2600 for a design based on the ORNL specifications for a large helium-cooled fast reactor.

Redesigning this reactor to decrease the specific power increases the fuel inventory and reduces the capital requirements. The variant allows an estimate of $\partial K/\partial F$, the tradeoff of capital for fuel in the helium-cooled reactor's production function. The increase in the fuel input is 337 kilograms over reactor lifetime as the specific power is reduced from 800 KW/Kg

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[26] Cf. the General Atomic Division, A Study of a Gas Cooled Fast Breeder Reactor, op.cit., p. 40, where it is argued that "raising the rate can lead to increased fuel cost if pursued too far. This arises because, although inventory charges are inversely proportional to rating, a component of the fuel fabrication cost increases with rating (because there is a limit to the power available per unit total length of fuel element, set by internal heat conduction, regardless of the coolant median) so somewhere there is an optimum as far as total fuel cost is concerned. The question is dealt with greater length in Section 12, where it is shown there is little or no economic incentive to use ratings much over some 800 KW/Kg." Ibid.

to 718 KW/Kg.[27] The reduction in capital is more difficult to assess. It appears from these studies of design variants, however, that the simplification of the core, and the reduction of helium temperature and pressure, as a result of 782-718/782 per cent power reduction, has the same effect on capital requirements as that per cent reduction in thermal megawatt capacity. The ORNL design variants for the helium-cooled fast reactor show an average reduction of 0.6 per cent in units of capital contained in the primary and secondary pressure vessels for each 1.0 per cent reduction in thermal capacity over the range 1200 to 2600 MW_t; on this basis, the 723 units of capital are reduced by 30 units, as a result of the 7.8 per cent power reduction. Then, from the reduction in specific power, capital is reduced by 30 units and fuel is increased by 337 units, or $\partial K/\partial F = -9/100$.

Most of the effects of the scale of output are shown by these design variants. A reduction in scale, from 2600 MW_t to 1317 MW_t as one variant in the ORNL study, brings capital in the pressure vessels alone down by approximately 50 per cent. Since the requirements for peripheral equipment -- purification and storage components -- are not changed substantially, then the number of units of capital is 565 rather than 693 at specific power of 718 KW/Kg. The effects on fuel inventory are shown by the fuel cycle design details for the 1317 MW_t helium fast reactor variant. The initial inventory is 2038 kilograms for this size of the general design at specific power

[27] The fuel cycle analyses show that a reduction from specific power of 782 KW/Kg to 718 KW/Kg involves a decrease in burnup from reduced core temperatures sufficient to require that lifetime fuel inventory increase to 7037 kilograms. This is an increase of 337 kilograms for a decrease in specific power of 64 KW/Kg. It is assumed that there is no further increase in inventory from a reduction in specific power from 800 to 782 Kg -- or that the inventory required at these two specific powers is roughly the same (as noted in the previous footnote).

of 646 KW/Kg; with an overall breeding ratio of 1.5 and a core lifetime of 3.6 years, the thirty-year requirement of fuel in present value terms is 3770 kilograms. The initial inventory is reduced to some amount close to 1789 kilograms, and the lifetime inventory to 3215 kilograms, by increasing specific power to 718 KW/Kg. Then this smaller reactor utilizes relatively less fuel over the inventory lifetime than does the larger reactor -- 3215 kilograms rather than 7037 kilograms -- or as noted in the General Atomic study, "fuel cycle cost [i.e., inventory per unit of capacity] does not vary greatly with reactor size, for cores of 5000 liters or larger."[28]

These approximations are enough to outline the dimensions of the production function. The tradeoff of the two inputs $\partial K/\partial F = -\psi' K_1/\beta' F_1 = -9/100$, which with $K_1 = 693$, $F_1 = 7037$, at $MW_1 = 2600$ and specific power of 718 KW/Kg is equal to $-\psi'(693)/\beta'(7037) = -9/100$. A second equation follows from the effects of scale; the ratios of the observations of inputs and of outputs in the production function are $\log(2600/1317) = \beta' \cdot \log(693/565) + \psi \log(7037/3215)$ which reduces to $2954 = 887\beta' + 3402\psi'$. Solving these two equations for $\beta' = .741$ and $\psi' = .674$, the first approximation to the production function for the helium-cooled fast breeder reactor appears to be

$$MW_t = K^{0.74} F^{0.67}$$

for thermal megawatts of capacity in terms of thirty years of capital and fuel requirements.

This production function can be placed next to those for the steam cooled fast breeder and the liquid metal fast breeder for contrast and

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[28] General Atomic Division, op.cit., p. 162.

similarity. The economies of scale, indicated by the sum of β' and ψ', are apparent but are less than from the liquid metal reactor ($\beta' + \psi' = 1.41$ for the gas cooled reactor, whereas $\beta' + \psi' = 1.68$ for the LMFBR). The productivity of capital is much less and that of fuel is somewhat greater than in the design based on liquid metal coolants (the marginal productivities of the two inputs are respectively $\partial MW_t/\partial K = \beta' MW_t/K$ and $\partial MW_t/\partial F = \psi' MW_t/F$). This is not surprising: the addition of small amounts of capital does not greatly add to capacity because the simple design requires duplication of almost all features to extend that capacity; but the addition of small amounts of fuel adds greatly to capacity because of the large breeding ratio in the helium design. There are differences, as well, between steam and helium cycles. Both show similar economies of scale ($\beta' + \psi' = 1.4$ in both production functions; it follows that a 1 per cent increase in both inputs implies a 1.4 per cent increase in capacity in the two cases). But the productivity of capital is less, and that of fuel is greater, in the helium fast breeder because of the presence of capital items that have been eliminated in the direct steam cycle, and because of the greater productivity to breed new fissile fuel than in the direct cycle.

Safety Characteristics of Steam-Cooled and Gas-Cooled Breeder Reactors

The safety of a fast breeder reactor concept can be considered in terms of variations of capacity MW_t from that called for in the design, where the variations follow from transient temperature changes in the reactor core. In the liquid metal cooled reactor designs, both *a priori* and experimental analyses have shown that, at certain very high fuel temperatures incursions

of more temperature or power will add to reactivity so as to lead to further temperature and reactivity increases. A departure from design temperature can lead to dis-equilibrium operation of the fuel core, with no self-adjustment back to equilibrium. The core can be shut down before it melts as a result of dis-equilibrium, but the shut-down itself is considered to take the operating value of MW_t to zero in some relevant time period. This behavior has been characterized, in the production function $MW_t = K^{\beta}F^{\psi}U$, in terms of $U = e^{-(T-T^*)(K/F)}$ as the productivity effects from any transient temperature T greater than design temperature T*. A similar characterization of the effects of temperature changes can be made for both the steam cooled and gas cooled reactor designs.

Experiments with reactivity and temperature changes point to the similarities in that aspect of behavior in steam and gas cooled cores, and to the contrast with that in liquid metal cooled cores. Temperature surges can be expected to occur infrequently in helium or steam during circulation of the coolant; for one, the occurrence of gas bubbles, as in liquid sodium, cannot lead to local hot spots which become temperature surges. But the coolant temperature will increase much more rapidly when there is an outage of gas pumping systems, because the lower adsorptive power of these coolants places central reliance on coolant flow to reduce core temperature. Then the concern in achieving reliability of gas and steam reactors is for temperature changes from voiding of the coolant from the core.

The first consideration is the fraction of the volume of the core in the two systems that has to be accounted for by coolant in order to achieve any given MW_t with given K and F. This fraction is more than 30 per cent greater in the steam reactor than in the gas reactor, for coolant temperature

conditions in the NDA and Oak Ridge design studies.[29] A second consideration is total pumping power required to circulate the coolant through the core; this is 60 per cent less in the steam than in the gas cooled reactor.[30] Given an outage in coolant system pumping, then, the gas reactor core is subject to a larger temperature increase: with more fuel per unit of coolant, and more reliance on pumping to remove heat, more of the cooling ability of the system is lost when there is no pumping power.

The temperature change reduces plant capability more in the helium than in the steam cooled reactor. For any given number of degrees of temperature change, there is more induced change in reactivity in the helium and plutonium core. The reactivity effect $\partial\rho/\partial T$ is parceled into ρ_1 and ρ_2 in both types of reactor; the value for $\partial\rho_1/\partial T$, the decrease in reactivity ρ from decreased coolant density at higher temperatures is ten times less in helium because its density does not vary greatly while $\partial\rho_2/\partial T$ is positive and large in both systems. Then the overall change in reactivity in the gas reactor is likely to be positive, since it is dominated by the value of $\partial\rho_2/\partial T$ rather than $\partial\rho_1/\partial T$ in this system. The gas reactor must be shut down more rapidly so that runaway expansive reactivity-to-temperature feedbacks do not take place.

Temperature changes from complete loss of pumping power, and thus from voiding of coolant in the core, are usually prevented from going too high by shutting down of the reactor. There has to be some way devised, in fact,

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[29] Cf. M. Dalle-Donne, "Comparison of Helium, Carbon Dioxide, and Steam as Coolants of 1000 Megawatt Electric Fast Reactor" (Euratom-gesselschaft fur Kernforschung mb II, 1965), Table I.

[30] Ibid, Table I.

for shutting down quickly enough to prevent melting of the core and, at the limit, causing a nuclear explosion. Present plans in both gas and steam reactors center on emergency routings which flood the core with water from the steam systems, which will reduce temperature but, by greatly reducing neutron leakage out of the core, will also increase reactivity. For safety -- or reliability of thermal megawatts of capacity -- there must be absorption of the additional neutrons, presumably by using core additives or "poisons" such as boron or gadolinium. But these additives affect the design level for production of thermal energy, so that a tradeoff -- yet to be specified -- has to be made of design thermal power for decreased probability of zero power for extended periods in the reactor lifetime.[31]

The tradeoffs, in the range of present values of specific power or temperature and pressure in the various cooling systems, are likely to establish that steam and liquid metal are approximately of the same level of reliability and that gas is somewhat less reliable. The temperature coefficients of reactivity are similar for the first two, and negative under most conditions; but those for gas-cooled reactor designs can be positive because of the lack of a "cancelling effect" from the smaller negative values of ρ from resonance absorption at higher temperatures (here shown as $\partial\rho_1/\partial T$, usually termed the "Doppler Effect").[32] Then the necessary shut-down time for the first two reactors is likely to be less, either because of less chance

[31] Cf., J. W. Hallam, R. K. Haling, P. Killian, and G. T. Peterson, "The Flood Safety of Steam and Gas Cooled Reactors," (U. S. Atomic Energy Commission, Contract AT(04-3)180, PA 13, 1965), Figure 1; cf., also, G. Sofer, et.al. op.cit., pp. 79-82 and General Atomic Division, op.cit. p. 72 et. seq.

[32] Cf., M. Dalle-Donne, op.cit., Table 4 for Doppler Coefficients and K. Cohen, and C. P. O'Neill, op.cit., p. 91 and Table 2 for Doppler Coefficients for the liquid metal fast breeder reactor shown in the General Electric Design Study.

of meltdown or because of less necessity to close down the reactor at $T > T^*$ to prevent core meltdown. This is shown by considering in $MW_t = K^{\beta}F^{\psi}U$ as that $U = e^{-x}$ with $x = (T-T^*)(K/F)$, the product of the temperature variant and capital/fuel ratio, as in the liquid metal production function. The lowest values of K/F in the range 1200 to 2500 MW_t are for the steam cooled reactor, the second lowest for the gas breeder, and the highest for the liquid metal breeder. To scale the value of x in terms of relative reliability, capital can be considered to be $K^* = 2K$ for the first two reactor types; then x is 30 per cent larger for the gas reactor and takes approximately the same value for the other two reactor types. The production function for the steam cooled breeder reactor is then, as a second approximation,

$$MW_t = K^{0.85}F^{0.58}e^{-(T-T^*)(K^*/F)}$$

while the gas cooled fast breeder is

$$MW_t = K^{0.74}F^{0.67}e^{-(T-T^*)(K^*/F)},$$

so that the contrast lies mostly in the values of K^*,F taken on by the reactor designs for given thermal megawatts of capacity.[33]

Summary and Evaluation

There are indications in this rough outline of production functions that the technologies of the different breeder reactors are quite different.

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[33]The probability of T departing from T^* varies for the three reactors, as well. Qualitatively, it seems larger for the occurrence of coolant loss and less for random but smaller differences in the gas and steam reactors than in the sodium reactor. This will be dealt with in productivity measurements.

This is shown both in individual observations of MW_t, K and F, and in the effects from increasing inputs K and F on the scale of output MW_t.

The design studies on which the functions have been based were not "optimized," in the sense of each showing the least-cost combination of capital and fuel for given capacity and factor prices, and for a separate technology; but they were in search of a best design in this sense and would have been discounted if far removed from the realms of expected least costs of electricity generation. They seem to indicate, at the least, minimum amounts of capital and fuel for given thermal capacity and given core temperatures. As such, the combinations of K and F chosen for 2500 to 2600 MW_t in the three types of breeders show differences in technology. The liquid metal breeder uses capital and fuel in the ratio of 1/8, while the gas cooled breeder has a ratio of 1/10 and the steam cooled breeder has the ratio of 1/20. The first system seems "capital using" at this level of capacity, in the sense that both more capital and a higher ratio of capital to fuel is required than in the other systems. The third system, for opposite reasons, seems to be "fuel using." The first is based on designs of complex heat transfer flows from a coolant that cannot come into contact with steam for the turbine generator, so that units of capital are added for both complexity and safety; but the last system is extremely simple in the design of units of capital while limited in the transfer of heat and in breeding in the steam environment.

For smaller scales of operation than 2500 to 2600 MW_t, the liquid metal fast breeder requires much greater amounts of capital and fuel than do the other two reactors. If capacity is reduced to half the 2500 MW_t level, then the allowed proportionate reduction in capital and fuel is limited to 15 per cent in the first reactor (given that the reduction in inputs R

equals $R^{\beta'+\psi'} = .5$, or $R^{1.68} = .5$, then R = .15). The reduction for the steam or gas cooled reactor is 22.5% of capital and fuel when moving back to this lower level of capacity (for $\beta + \psi \sim 1.4$, then R = .225). This again is a result of "capital using": of the necessity for certain units of capital at all levels of output in the liquid metal reactor, as compared to the possibility of "scaling down" the units of capital in the gas and steam reactors.

These are initial impressions. A study of the behavior of these production functions given expected ranges of factor prices can indicate the differences in technologies more clearly. The terms of reference are the costs of producing output -- of additional capacity to produce thermal energy -- from the different systems and the values of these additions to capacity.

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